Polar Convexity - Applications

Hristo Sendov[1]

[1]Department of Statistical and Actuarial Sciences, Department of Mathematics, The University of Western Ontario, London, Canada

Abstract

Let $\hat{\mathbb{R}}^n$ be the one point compactification of $\mathbb{R}^n$ obtained by adding a point at infinity. We say that a subset $A \subseteq \hat{\mathbb{R}}^n$ is $\mathbf{u}$-*convex* if for every pair of points $\mathbf{z}_1, \mathbf{z}_2 \in A$, the arc of the unique circle through $\mathbf{u}, \mathbf{z}_1$ and $\mathbf{z}_2$, from $\mathbf{z}_1$ to $\mathbf{z}_2$ and not containing $\mathbf{u}$, is contained in A. In this case, we call $\mathbf{u}$ a pole of A. When the pole $\mathbf{u}$ approaches infinity, $\mathbf{u}$-convex sets become convex in the classical sense.

In this talk I will give several applications of polar convexity to classical polynomial problems. I will show a stronger version of the Gauss-Lukas theorem, Laguerre theorem, Grace theorem, Schur-Szego composition theorem, and a theorem by Hormander for multivariate polynomials. Two continuous selection theorems will also be presented. We will show a polar convexity analogues of the Cellina's approximate selection theorem and Michaels exact selection theorem.

Parts of this talk are based on joint work with Blagovest Sendov, Shubhankar Bhatt, and Pando Georgiev.